A mathematical model of the groundwood process: Part 2: Log grinding

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Abstract: Part 1 presented a mathematical model of the Groundwood process assuming that the morphology of the wood charge is a fundamental variable. This assumption was tested in the laboratory by grinding a model log. The data were found to have a good fit to the grinding model. At constant power, this morphology causes the specific energy to be very low at the start and end of the stroke, resulting in high Freeeness and %Shives.

In the laboratory, all of the model parameters are effectively constant throughout the stroke. As a result, constant inputs of normal force and stone speed result in constant values through the stroke length of all of the outputs.

In the mill, the literature indicates that this is not the case. Collicut [5] stated in 1951 that the wood/stone contact surface changes within-stroke. In 1964, Affleck [4] reported the wood-to-stone contact area to be an important cause of within-stroke variation. He found that within-stroke freeness variation, at the same average freeness, was 3 times greater when grinding logs compared with grinding sawn wood blocks. He also found that the freeness variability increased with log diameter. Affleck also found that 400 strokes were required to produce a given amount of pulp using square sawn log (slabs) compared with 520 strokes using round logs.

Wood Charge Morphology

The most likely underlying cause of these findings is that the wood contact fraction depends on the morphology of the log charge. The wood contact fraction becomes a profile \( \Phi_W (x) \) through the stroke. It is worth noting, from a morphological viewpoint, that grinding is among the closest natural product feedstock.

The geometry of a single log is simple and deterministic. In addition, it is straightforward to adapt a laboratory grinder to grind a sample with the geometrical features of a single log.

SINGLE LOG MODEL

Log Morphology

A single log, as a first approximation, is a “right” cylinder of constant diameter \( D \). When this cylinder is ground radially, the wood surface area pressing against the stone, at any position during the stroke, approximates a rectangle with width equal to the log length and height equal to a cord of the circle of diameter \( D \). This cord is zero at the start of stroke; increases to the log diameter \( D \) halfway through the stroke and then reduces to zero at the end of the stroke. The normalized length of this cord is given by:

\[
\Phi_W (x) = \sqrt{4 \left( \frac{x}{D} \right) - 4 \left( \frac{x}{D} \right)^2} 
\]  

Grinding of a Single Log

The general model [1] when the wood fraction is a profile \( \Phi_W (x) \) at any position \( x \) during the stroke is:

\[
V_G (x) = 100 \left( K_G \cdot V_S \right) p_G^2 (x) 
\]

Where:

\( V_G (x) \) is the grinding speed profile

\( K_G \) is the “Coefficient of Grinding”

\( V_S \) is the peripheral speed of the stone

\( p_G (x) \) is the “true grinding pressure” profile

\[
p_G (x) = N(x)/A_G (x) = N(x)/(A_P \Phi_W (x) \Phi_S) 
\]

Where:

\( N(x) \) is the profile of the normal force

\( A_G (x) \) is the profile of the “true” grinding area

\( A_P \) is the cross section area of the pocket.

The following parameters are essentially constant during a single stroke: \( K_G, V_S, A_P, \Phi_S \). The two variables are \( V_G \) and \( N \). The experimental apparatus was such that automatic control of grinding speed \( (V_G) \) was more effective than control of force \( (N) \). Grinding speed was therefore selected as a constant input and force as the variable output. The wood fraction profile, \( \Phi_W (x) \) was modeled by equation [1].

Re-arrangement of equations 2 and 3, at constant grinding speed, shows that \( N(x) \) is directly proportional to \( \Phi_W (x) \):

\[
N(x) = \kappa \Phi_W (x) 
\]

where:

\( \kappa = \sqrt{100 K_G V_S} \)

EXPERIMENTAL

Wood raw material

Fresh spruce (Picea Abies (L.) Karst.) was utilized in this work. After felling, the logs were sawn into wood discs 39 mm thick. These were sorted into three different categories according to diameter (stump \( \geq \Phi 200 \text{ mm}, \) mid-height \( \geq \Phi 150 \text{ mm}, \) top \( \geq \Phi 100 \text{ mm} \)). The stump discs were further sawn into 39 mm wide blocks for the laboratory grinding experiments, Fig. 1. Two blocks from the middle of the discs were considered as heartwood, while the others represented sapwood. Some sapwood blocks were further sawn into 39mm*39mm*39mm cubes which were then turned round in a lathe. Initial trials showed that...
the cylindrical sample could “roll”, thus voiding the assumption of radial grinding. To avoid “rolling”, these round cylinders were sawn into “semi-cylinders.” The heartwood blocks had an average moisture content (MC) of 29%, while the sapwood blocks had 57.9%. Only sapwood blocks were used in the experiments, because the sapwood MC represented that of fresh wood. Only uniform, knot-free blocks were selected for the experiments to minimize deviation in the results.

**Grinding experiments**

The SGW-grinding experiments were performed with the recently modernized laboratory grinder of the Laboratory of Pulping Technology, Åbo Akademi University [3]. A ceramic, commercial-type pulpstone was used in the experiments, because the sapwood MC represented that of fresh wood. Only uniform, knot-free blocks were selected for the experiments to minimize deviation in the results.

The good agreement of the experiments with the model provides convincing evidence that the morphology of the wood charge is a critical component in relating mill-scale grinding to the laboratory. The high degree of non-linearity makes this approach especially useful. This proof that the morphology of the log charge is a fundamental variable in the grinding process permits its extrapolation to the much more complex morphology encountered in the mill case.

In the case of a single log, the stroke-average wood fraction is \( \frac{\pi D^2}{4} \). In the mill, Affleck [4] compared the grinding of logs with sawn blocks and found a value of 400/520 = 0.77. It is not surprising that these two numbers are quite close. The laboratory mechanism of packing a single circle into a square is very close to that of packing a batch of circles (logs) of random diameter into a (pocket) rectangle of constant height and variable (stroke) length.

**Experimental Results**

The experimental results are shown in Figs. 2 and 3. The model is given by equation [4].

The non-linear trend of the experimental data closely follows the model. The superimposed waveform in the force data is due to oscillation of the automatic speed control system causing a phase shift between the speed and force.

Figure 4 shows a straight line between the tangential and normal forces. Hence \( K_F \), the coefficient of friction is constant, despite the wide variations in grinding area and pressure.

With constant \( K_F \), the model shows that, at constant force, the specific energy is proportional to \( \Phi_W(x) \). The wood sample was too small to be able to perform quality tests, however the model [1] shows that the specific energy profile results in very high Freeness and %Shives both early and late in the stroke, Fig. 5.

**DISCUSSION**

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The constancy of the coefficient of friction \( K_F \) through the stroke, despite a very wide variation of the grinding area, simplifies prediction of the specific energy and hence pulp quality [1]. The very low specific energy at the start and end of the stroke means that both the Freeness and %Shives will be very high at these positions in the stroke.

It is worth noting that the general model [1] permits “on-line” estimation of \( \Phi_W(x) \) and hence specific energy at each point \( x \), through the length of each stroke, thus providing the theoretical basis [2] for within-stroke, model-based, rate, specific energy and pulp quality control. The inherent non-linearity of the model can be used to explain several poorly understood phenomena related to blending and sampling of pulp.
CONCLUSIONS
A general, input-output model of wood grinding has been extended to include the morphology of the wood charge, in particular to a single log, approximated by a cylinder. Good agreement between the model and experimental data was found. This provides the proof needed to make the assumption that the much more complex morphology of the wood charge in mill grinding is a fundamental variable.

The coefficient of friction ($K_f$) was shown to be unaffected by the wood contact fraction and specific grinding pressure over its complete range. This simplifies the modeling of specific energy and hence pulp quality.

The mechanistic, multivariable, nonlinear nature of the model makes it useful for extrapolation from laboratory grinding to mill scale grinding. It should have value, for example, in the design of experiments aimed at understanding the effect of various stone design and operating factors on the grinding and friction coefficients.

The model provides a sound theoretical basis for on-line estimation in the mill of the wood fraction profile and hence specific energy and pulp (“soft-sensor”) quality during grinding. This enables the use of within-stroke model-reference control in order to optimize the within-stroke operation in terms of quality variability, cost (specific energy usage) and throughput.

LITERATURE:

Résumé: La Parte 1 présentait un modèle mathématique du procédé de fabrication de la pâte mécanique, en présumant que la morphologie de la charge de bois est une variable fondamentale. Cette hypothèse a fait l’objet d’essais en laboratoire où une bille modèle a été défrisée. Les données correspondaient bien à celles du modèle de défrassage. À puissance constante, cette morphologie rend l’énergie spécifique très faible au début et à la fin de la course, ce qui entraîne une teneur en bûchettes et un indice d’égouttage élevés.


Keywords: MATHEMATICAL MODELS, PROCESS CONTROL, OPTIMIZATION, GROUND WOOD, GRINDING, LOGS, PRESSURE, COEFFICIENT OF FRICTION.