Application of wavelet transformation for paper anisotropy

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Abstract: A novel method for evaluating the anisotropy of paper formation is introduced and is applied to a range of laboratory paper samples. This method is based on the application of one-dimensional wavelet transformation to quantify variability in the local grammage of paper. The local grammage map is obtained by beta-radiography. Since a lower value of wavelet energy is resulted in the direction that fibres orient, the ratio of the wavelet energy in the cross-machine direction to that in the machine direction can be used as an index to quantify the extent of structural anisotropy. Using this approach, the angular distribution of wavelet energy which corresponds to the fibre orientation distribution is calculated. For comparison, a technique based on the use of power spectrum is also employed to quantify the anisotropy of these paper samples.

During the papermaking process, paper structure tends to align in the machine direction (MD) rather than the cross-machine direction (CD). This preferential orientation is a result of the accelerating forces that are exerted on the fibres in the headbox and on the forming section of the paper machine [1]. The drying shrinkage and wet straining of the paper web [2] also contributes to this structural anisotropic behavior. The study of paper anisotropy is significant as it can be a useful tool for characterizing and monitoring paper quality in the manufacturing process, and understanding its impact on the end-use properties of paper.

The extent of anisotropy in paper is usually determined by the degree of fibre orientation in the sheet [3]. Methods that are available in determining the extent and direction of fibre orientation usually belong to one of the following two categories: direct measurement technique, where direct observation of the arrangement of fibres in the paper network is made; and indirect technique, where the degree of anisotropy and orientation of structures is indirectly determined, often through evaluating the ratio of mechanical or optical properties in the machine direction to that in the cross-machine direction [4].

One of the direct techniques available involves manually determining the orientation and the length of the coloured fibres in the fibre web with the help of an optical microscope. This method is subject to the problem of reproducibility introduced by the use of different observers. Another approach that makes use of digital image analysis instead of observers is able to avoid subjectivity in the result.

The degree of anisotropy of paper can also be reflected by its degree of anisotropy in mechanical and optical properties. Zummeren et al [5] used an ultrasonic method to measure the tensile stiffness and elastic properties in paper. Multiple sensors were used to determine the speed of travel of the ultrasonic signals in different directions, which is related to the elastic property of the sample. The ratio of maximum to minimum elastic stiffness is defined as tensile stiffness index and is an indication of the anisotropy of the sheet. In terms of optical properties, Koukoulas and Jordan [6] reported the determination of the specular and diffuse components of gloss through the use of a goniophotometer, and this was proposed to be related to surface fibre anisotropy. In their work, the anisotropy ratio was defined as the ratio of the reflectance measured in the machine direction to that in the cross-machine direction. Another instrument explored by Drouin et al [7] was based on the use of far infrared radiation, where anisotropy was determined from the dichroic ratio obtained from the dichroic and emission spectra. Fiadeiro et al [4] proposed a new optical method to determine fibre anisotropy through the use of laser diffraction analysis.

The spatial information carried in the measurement of the paper properties can be parameterized with the use of a statistical tool. Methods that have been developed include the use of coefficient of variation of grammage, defined as the variance divided by the mean local grammage of the sample. This was used by Schaffnit et al to determine the formation anisotropy ratio, which was simplified to the ratio of the variance of grammage in a certain zone in the machine direction to that in the cross-machine direction, because analysis in both directions shares the same mean grammage in the zone [8].

Other statistical tools might express the variability in the data as a function of scale or resolution. Determination of variance in data as a function of zone size has been conducted [9]. Schacanski et al [10] applied the Prewitt operator based on the theory of texture image analysis to evaluate the formation anisotropy of paper. Another approach is the Fourier analysis which results in obtaining the power and wavelength spectrum. Yuhara et al demonstrated the use of normalized angular distribution of the power spectrum of mass distribution to simulate the distribution of fibre orientation [11].

In this study, an approach based on the appli-
cation of wavelet transformation is used to evaluate the anisotropy of grammage in paper. This method, introduced by Keller et al to study paper variability on simulated formation images [12, 13], has the capability to analyze non-stationary data and extract spatially localized features in the data. Using this technique, the anisotropy of several laboratory paper samples is determined. These results are compared with those obtained from the use of Fourier transform and Prewitt operator.

**WAVELET TRANSFORMATION ANALYSIS**

Wavelet transform analysis is a useful tool for analyzing non-stationary data, where its variance is spatially localized. Mathematically, wavelet transform is the convolution of the function being investigated with the wavelet function and is given by the following equation:

\[
W(a,b) = \int f(x) \frac{1}{a} \psi \left( \frac{x - b}{a} \right) dx
\]

where \(W(a,b)\) is a set of wavelet transform coefficients; \(f(x)\) is the function being analyzed; \(\psi(x)\) is the mother wavelet, a localized wavelike function that satisfies certain mathematical criteria [14]; \(a\) and \(b\) are the scale and translation parameters, respectively. Wavelet energy, \(E_W\), is defined as the squared magnitude of the wavelet transform coefficients:

\[
E_W(a,b) = |W(a,b)|^2
\]

In the analysis, the choice of the mother wavelet and the range of scale with which the transform is carried out depend on the application. In the case of this study, wavelet transform is used to analyze the structural variability in paper. The presence of variability leads to abrupt changes in values of the measurement, which create local maxima and minima in the data. The Marr wavelet, also known as the Mexican hat wavelet, is a wavelet function that is capable of extracting the local maxima and minima features in the data. It is the second derivative of the Gaussian function, defined as

\[
\psi(x) = (x^2 - 1)e^{-x^2/2}
\]

This is the mother wavelet adopted in the analysis in this study. The resolution of the analysis is determined by the scale parameter of the wavelet with which the transform is carried out. The first part of this work sets the scale parameter to be equivalent to 30 μm. This value is chosen because it is comparable with the dimension of fibre width so that fibre orientation of the paper samples can be determined [11]. The second part of this work conducts the wavelet analysis using other different resolutions and examines the effect of scale on the study of paper anisotropy.

**Determination of anisotropy ratio**

Beta-radiographs and local grammage maps of paper samples are obtained using the procedures described in Farnood et al [15] for analysis. The grammage map of each sample is digitally rotated every 4° for a total of 360°. Each row of pixels in each rotated image is treated as a 1-D function and is convoluted with the wavelet function to obtain the wavelet energy. This is done for all the rows of the grammage image to obtain a matrix of wavelet energy that corresponds to the degree of orientation of the samples. This is illustrated in the polar plot, Fig. 2. The plot of angular wavelet energy distribution for the most isotropic sample (mdl jj4a1) is elliptical in shape, while those for the more anisotropic samples (pxxe series) acquire the shape of a peanut, with the ratio of the longitudinal to latitudinal length of the peanut figure, which corresponds to the anisotropy ratio, increasing with the orientation of the samples.

Anisotropy ratios resulted from wavelet analysis are found to increase with the degree of orientation of the samples. This is illustrated in the polar plot, Fig. 2. The plot of angular wavelet energy distribution for the most isotropic sample (mdl jj4a1) is elliptical in shape, while those for the more anisotropic samples (pxxe series) acquire the shape of a peanut, with the ratio of the longitudinal to latitudinal length of the peanut figure, which corresponds to the anisotropy ratio, increasing with the orientation of the samples.

**RESULTS**

The anisotropy of a series of increasingly oriented samples, listed in Table I and ranked down the column in order of increasing anisotropy, are examined. This set of papers has been studied by Schacanski et al and more information on these samples can be found in [10]. The different degrees of anisotropy in these samples can be observed in the images of the samples’ grammage maps, Fig. 1.

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The wavelet anisotropy ratios are compared with the results obtained from Fourier analysis [11] and a texture image analysis method [10]. In Fourier analysis, as in the approach that is used in wavelet analysis, the power of each row of pixels of the grammage image is found using a frequency range of 1 - 4 mm. The average of the power found for all rows of the image...
is normalized after the above procedures are carried out for all angular orientations to obtain the angular power distribution of the image, $E_p(\theta)$. The anisotropy ratio, $R_p$, is found from the power distribution using the same method described above for the wavelet analysis, and is listed in Table II. In the texture image analysis conducted by Schacanski et al [10], the Pre-witt operator is used to evaluate the eccentricity parameter, $e^2$, which corresponds to the anisotropy of these papers. Their results for this same set of samples are listed in Table II for reference.

Linear regression analysis, Fig. 3, shows a strong correlation between the anisotropy ratio obtained from wavelet analysis and that from Fourier analysis ($r^2$ of 0.91) with the data points scattered around the 45°-line.

The angular normalized power distribution, $E_p(\theta)$, for the samples is plotted against the rotation angle in Fig. 4. It can be observed that, while the shapes of the curves have the same trend as those shown in Fig. 2 from wavelet analysis, the smoothness of the plots differs significantly. This difference may be attributed to the highly localized nature of Fourier analysis in the frequency domain [13], which explains its greater sensitivity to abrupt variations in a signal. In other words, Fourier transform is based on infinitely long sine waves, which means that a large number of Fourier terms are required to express local features in the data set. In comparison, wavelet functions are finite in length, and therefore wavelet transformation is more effective for extracting localized features and variations in a signal.

**Effect of Scales on Anisotropy**

The wavelet technique is an effective tool for conducting multi-resolution analysis. The results presented in this paper thus far have focused on the determination of fibre orientation in the paper samples, with the size of fibre width being the resolution of the study. Nonetheless, the degree of anisotropy of paper structures at larger scales can also be obtained using the same procedure as described earlier.

Table III shows the result of using various wavelet scales, ranging from 30 µm (comparable to fibre width) to 3 mm (comparable to floc size), to determine the anisotropy of paper. For all the samples, it is found that, as the scale of study increases, the anisotropy index decreases. At the level of fibre width (scale of 30 µm), the anisotropic feature of the paper samples is the most prominent, reflecting the preferential orientation of fibres in the samples. At the scale of fibre length (scale $\sim 600-1200$ µm), some degree of anisotropy in the paper samples is evident as the anisotropy ratio ($R_p$) reaches as high as 1.28, indicating that small flocs are somewhat oriented in the samples. However, at the larger scales of 2.4 mm to 3 mm, no orientation is detected as $R_p$ is nearly 1 for all samples. This finding indicates that larger flocs may be considered to be randomly
oriented. To further illustrate this result, the effect of scale on the polar plot of wavelet energy for the most oriented paper sample (pxxe50c1) is shown in Fig. 5. The change of shape of the polar plot as a function of scale can be observed — small scales result in a peanut shape for the curve, while large scales result in plots that are close to an elliptical shape.

**CONCLUSION**

A method based on the use of one-dimensional wavelet transformation is introduced in this paper to determine the formation anisotropy of paper. Wavelet transform analysis yields wavelet energy that represents the structural variability of paper. The transformation can be performed along different directions to obtain an angular distribution of variability in the sample. Paper anisotropy can be determined by evaluating the ratio of the wavelet energy in the cross direction to that in the machine direction because structural variability is lower in the direction that fibres orient.

Using the technique developed, the formation anisotropy of a series of increasingly oriented laboratory samples is determined. The expected result, where anisotropy increases with the degree of orientation in paper, is obtained. A good correlation is found between these results and the ones obtained from the use of Fourier transform analysis.

The effect of scale on paper anisotropy is also examined. It is found that, as scale increases, the anisotropy ratio of paper decreases, reflecting the higher degree of fibre orientation compared to floc orientation.

The wavelet transform technique developed can be applied to data sets other than grammage maps, such as gloss and topography maps, to study the anisotropy in optical properties and surface texture of paper.

**LITERATURE**


Résumé: Nous avons élaboré une méthode numérique portant sur la compressibilité du papier afin de déterminer la relation entre les propriétés mécaniques du papier et les caractéristiques de la fibre. L’étude de la compression du papier est utile, parce qu’une meilleure compréhension de ce mécanisme peut aider à améliorer et à contrôler les processus de fabrication du papier et d’impression lorsque le papier est compressé. La plus grande partie des connaissances acquis jusqu’à maintenant dans ce domaine l’ont été de manière empirique, et les modèles numériques disponibles ne permettent pas de faire le lien entre les propriétés des fibres et la dynamique de la déformation du papier dans le sens de l’épaisseur dans des échelles temporelles d’une pertinence commerciale.

Pour résoudre ce problème, nous avons ici étudié, sur le plan numérique, la compression de réseaux tridimensionnels. Nous avons élaboré un programme d’ordinateur afin de créer un réseau de fibres en trois dimensions qui simule la structure qu’imprime. La compression dynamique de ce réseau de fibres est alors simulée à l’aide d’une méthode explicite d’analyse par éléments finis utilisant le logiciel commercial ANSYS LS-DYNA. Ce logiciel permet la détermination dynamique des points de contact fibre-fibre. Grâce à cette méthode, les fibres sont modélisées comme des éléments d’un faisceau élastique-plastique et le comportement transitoire du faisceau de fibre 3D en mode compression est analysé. À l’aide de cette technique, il est possible d’étudier les effets de divers paramètres d’exploitation, y compris les caractéristiques de la fibre (souplesse et dimensions des fibres), la structure du papier (grammage et porosité) et, dans le cas du calandrage, la charge des lignes et les propriétés physiques de la couverture souple sur le processus de compression du papier.


**Keywords:** ANISOTROPY, FORMATION, MEASUREMENT, VARIABILITY, BASIS WEIGHT, RADIOGRAPHY, WAVES, PAPER.